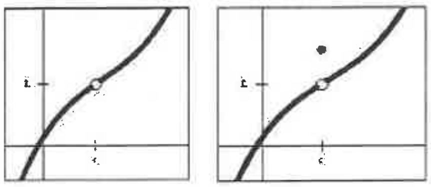
Types of Discontinuities

REMOVABLE

- $f(c)$ may or may not be defined

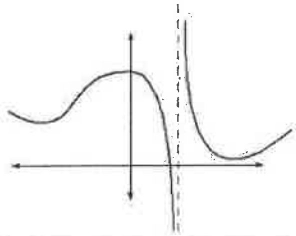
$\lim_{x\to c} f(x)$ exists but $\lim_{x\to c} f(x) \neq f(c)$



INFINITE

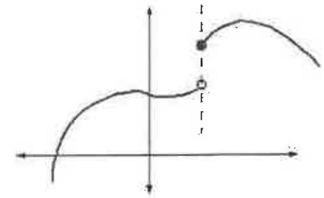
$\lim_{x\to c} f(x) = \pm\infty$

$f(x)$ is unbounded near $x = c$



JUMP

$\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$



*Note: Both infinite and jump discontinuities are considered "non-removable"

For each function identify the type of each discontinuity and where it is located.

1. $f(x) = \dfrac{1}{x+2}$

f(x) has an infinite disc @ x=-2.

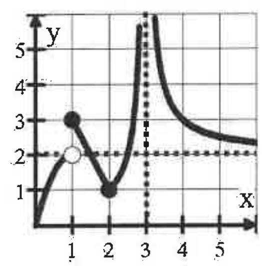
2. $f(x) = \dfrac{x-2}{x^2-4} = \dfrac{x-2}{(x+2)(x-2)}$

f(x) has a removable disc @ x=2 & an infinite disc @ x=-2.

3. $f(x) = \dfrac{x-2}{|x-2|} = \begin{cases} -1 & x<2 \\ 1 & x>2 \end{cases}$

f(x) has a jump disc @ x=2.

4.



jump disc @ x=1

infinite disc @ x=3.

Given the function $f(x)$, determine the intervals on which $f(x)$ is continuous.

5. $f(x) = \dfrac{x-7}{x^2-9x+14} = \dfrac{x-7}{(x-7)(x-2)}$

f(x) is cont on $(-\infty, 2) \cup (2, 7) \cup (7, \infty)$

6. $h(x) = \dfrac{\sqrt{x-3}}{x-5}$ D: $[3, 5) \cup (5, \infty)$

f(x) is cont on $[3, 5) \cup (5, \infty)$

7. $f(x) = \begin{cases} 3^x, & x \le -1 \\ \dfrac{2x+3}{x+4}, & -1 < x < 0 \\ x^2+2x, & x > 0 \end{cases}$

x = -1

$\lim_{x\to -1^-} 3^x = \frac{1}{3}$

$\lim_{x\to -1^+} \frac{2x+3}{x+4} = \frac{1}{3}$

x = 0

$\lim_{x\to 0^-} \frac{2x+3}{x+4} = \frac{3}{4}$

$\lim_{x\to 0^+} (x^2+2x) = 0$

f(x) is cont on $(-\infty, 0) \cup (0, \infty)$.

Removing a Discontinuity

8. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-9}{x-3}$ when $x\neq 3$, then $f(3)=$ $\lim_{x\to 3}\dfrac{x^2-9}{x-3}$ $\lim_{x\to 3}(x^2-9)=0$ $\lim_{x\to 3}(x-3)=0$ $\lim_{x\to 3}\dfrac{(x+3)(x-3)}{x-3}$ $f(x)=\begin{cases}\dfrac{x^2-9}{x-3} & x\neq 3\\ 6, & x=3\end{cases}$ $\lim_{x\to 3}(x+3)=6$	9. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2+8x-20}{x+10}$ when $x\neq -10$, then $f(-10)=$ $\lim_{x\to -10}\dfrac{x^2+8x-20}{x+10}$ $\lim_{x\to -10}(x^2+8x-20)=0$ $\lim_{x\to -10}(x+10)=0$ $\lim_{x\to -10}\dfrac{(x+10)(x-2)}{x+10}$ $\lim_{x\to -10}(x-2)=-12$ $f(x)=\begin{cases}\dfrac{x^2+8x-20}{x+10}, & x\neq -10\\ -12, & x=-10\end{cases}$
10. Let f be the function defined by $f(x)=\begin{cases}\dfrac{x^2-2x-15}{x-5}, & x\neq 5\\ k, & x=5\end{cases}$. For what value of k is f continuous at $x=5$? $\lim_{x\to 5}\dfrac{x^2-2x-15}{x-5}$ $\lim_{x\to 5}(x^2-2x-15)=0$ $\lim_{x\to 5}(x-5)=0$ $\lim_{x\to 5}\dfrac{(x-5)(x+3)}{x-5}$ $\lim_{x\to 5}(x+3)=8$ $\boxed{k=8}$	11. Let f be the function defined by $f(x)=\begin{cases}\dfrac{x^2+5x+4}{b(x+1)}, & x\neq -1\\ b, & x=-1\end{cases}$. For what value of b is f continuous at $x=-1$? Since $\lim_{x\to -1}(x^2+5x+4)=0$ & $\lim_{x\to -1}b(x+1)=0$ $\lim_{x\to -1}\dfrac{x^2+5x+4}{b(x+1)}=\lim_{x\to -1}\dfrac{(x+4)(x+1)}{b(x+1)}$ $=\dfrac{3}{b}$ $\dfrac{3}{b}=b\to b^2=3$ $\boxed{b=\pm\sqrt{3}}$
12. Let f be the function defined by $f(x)=\begin{cases}\dfrac{x^2-16}{\sqrt{x}-2}, & x\neq 4\\ w, & x=4\end{cases}$. For what value of w is f continuous at $x=-1$? $\lim_{x\to 4}\dfrac{x^2-16}{\sqrt{x}-2}=\lim_{x\to 4}\dfrac{(x+4)(x-4)}{\sqrt{x}-2}$ $=\lim_{x\to 4}\dfrac{(x+4)(\sqrt{x}+2)(\sqrt{x}-2)}{\sqrt{x}-2}$ $=\lim_{x\to 4}(x+4)(\sqrt{x}+2)=32$ $\boxed{w=32}$	13. Let f be the function defined by $f(x)=\dfrac{x^2-(k+2)x+k}{x-k}$ For what value of k wil the function have a removable discontinuity at $x=k$? $x-k=0$ $x^2-(k+2)x+k=0$ $x=k$ $k^2-(k+2)k+k=0$ $k^2-k^2-2k+k=0$ $-k=0$ $k=0$ $f(x)=\dfrac{x^2-2x}{x}$ removable disc @ $x=0$